the following equations for A and B,

$$B = [(P/2\pi r_0)\alpha_{x2}(a) + q_0\alpha_{52}(a)]/\alpha_{32}(a)$$
$$A = [(P/2\pi r_0)\alpha_{42}(a) + q_0\alpha_{52}(a)]^{w_m(a)}/\alpha_{32}(a) \quad (P/2\pi r_0)w_Q(a) - q_0 w_l(a) \tag{5}$$

provided $k \neq 1$.

The expressions in Eq. (5) may now be obtained by direct substitution from Table 2. The procedure is straightforward and the example, will not be persued further.

Summary

Starting functions for the solution of axisymmetric orthotropic annular plates are given along with their stress resultants. An example indicating their use was presented. It was shown that the arbitrary constants used in starting the problem could be obtained from the outer boundary condition. It should be pointed out that w_l may also be used for the case of variable loading. A similar set of starting functions may also be devised for other problems, for example, thick plates.

Reference

[1] Lekhnitskii, S. G., *Anisotropic Plates*, 2nd ed., Gordon and Breach, New York, 1968, p. 370.

Upwash Interference on a Jet Flap in Slotted Tunnels

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Introduction

THE V/STOL models used in the development of a wind-tunnel wall configuration for V/STOL testing should include a spectrum of models from the high disk loading (e.g., lifting jet, lifting fan, and rotor) to the low disk loading models (e.g., jet-flat wing). The high disk loading model in slotted-wall tunnels has been investigated by the author in Refs. 1–3. The interference of a jet-flap wing in slotted-wall tunnels will be considered as part of the program in the present study. A detailed mathematical treatment of this problem has not yet become available,[4] although some qualitative discussions are presented in Ref. 4 for the closed-wall tunnel.

This paper presents the upwash interference on a two-dimensional jet-flap wing in a slotted-wall tunnel. The formulation is based on the small disturbance theory and the linearized model of the jet-flap wing as derived by Spence.[5] An analytical solution is developed for the upwash interference and some numerical results are shown in the graphical form.

Analysis

The field equation of an inviscid, irrotational fluid for incompressible flow in terms of the perturbation velocity potential φ in Cartesian coordinates (Fig. 1) is

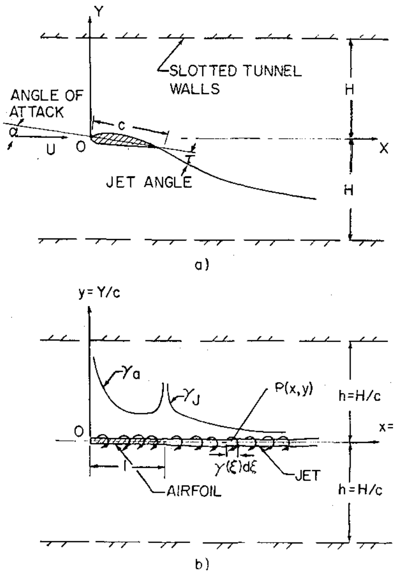

Fig. 1 Jet-flap wing in a slotted-wall wind tunnel and its mathematical model.

$$[(\partial^2/\partial X^2) + (\partial^2/\partial Y^2)]\varphi = 0 \tag{1}$$

The boundary conditions for the slotted walls using an equivalent homogeneous boundary condition[1] are expressed as

$$\varphi \pm (K\partial\varphi/\partial Y) = 0,\ Y = \pm H \tag{2}$$

where K is related to the wall porosity a/l as $K = (l/\pi)\ln[\csc(\pi a/2l)]$. The slot parameter is introduced as $P = (1 + K/H)^{-1}$ where the value of $P = 0$ corresponds to a closed wall and $P = 1$ to an open wall. Note that the geometric slot constant, K, in Eq. (2) may also be related to the wall porosity a/l by $K = [(l - a)/2]\tan[\pi(1 - a/l)/2]$ as in Ref. 8. This gives a good correlation between theory and experiment in some slotted wind tunnel investigations.

The linearity of the field equation and its boundary condition permits the perturbation potential to be separated into two parts as $\varphi = \varphi_m + \varphi_i$ where φ_m is the disturbance potential caused by the model in free air and φ_i is the interference potential induced by the tunnel walls.

The two-dimensional jet-flap wing located at the centerline in a slotted-wall wind tunnel is shown in Fig. 1a. The linearized model of a jet-flap wing shown in Fig. 1b has been analyzed by the small disturbance theory.[5] The airfoil and jet are represented by the vorticity distribution γ which

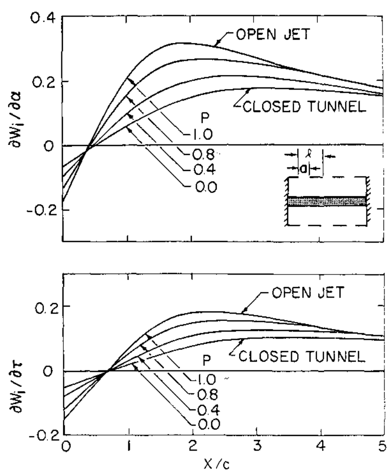

Fig. 2 Upwash interference for a jet-flap wing with C_J = 2.0 in a slotted tunnel with various wall porosities at H/c = 1.5.

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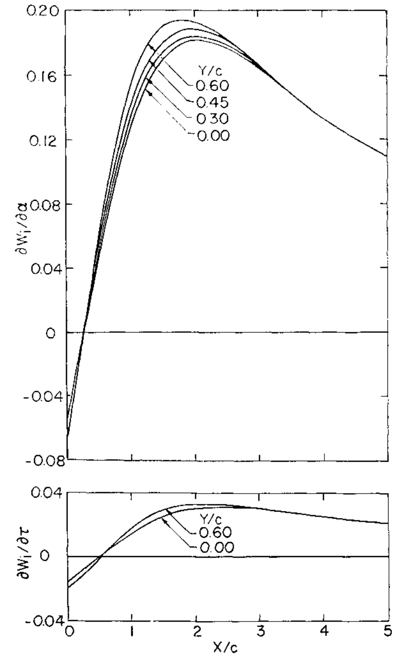

Fig. 3 Upwash interference distribution for a jet - flap wing with C_J = 0.1 in a slotted tunnel with H/c = 1.5 and P = 0.6.

may be expressed as follows:

for the airfoil

$$0 < X/c = x < 1, \quad x = \tfrac{1}{2}(1 - \cos\theta)$$

$$\frac{\gamma_a(x)}{U} = \frac{2\tau}{x^{3/2}}\left[-\frac{1}{\pi}\log(1-x) + A_0\left(1 - \cos\frac{\theta}{2}\right) + \left(\sum_{n=1}^{N-1} A_n \tan\frac{\theta}{4}\right)^{2n}\right] + \frac{2\alpha}{x^{3/2}}\left[x(1-x)^{1/2} + B_0\left(1-\cos\frac{\theta}{2}\right) + \sum_{n=1}^{N-1} B_n\left(\tan\frac{\theta}{4}\right)^{2n}\right] \quad (3)$$

for the jet

$$1 < x < \infty, \qquad x = \sec^2(\psi/2)$$

$$\frac{\gamma_j(x)}{U} = \tau\left[-\frac{4}{\pi}\cos^3\frac{\psi}{2}\log\left(\tan\frac{\psi}{2}\right) + 2\cos^3\frac{\psi}{2} \times \sum_{n=1}^{N-1} A_n \cos n\psi\right] + \alpha\left[2\cos^3\frac{\psi}{2}\sum_{n=1}^{N-1} B_n \cos n\psi\right] \quad (4)$$

where A_0, A_1 . . . and B_0, B_1 . . . are functions of the jet momentum coefficient C_J given in Ref. 5.

The disturbance potential of a jet-flap wing may be obtained from Spence's model with all lengths nondimensionalized by the wing chord c as

$$\varphi_m = \frac{-1}{2\pi}\int_{-\infty}^{\infty}\gamma(\xi)d\xi \tan^{-1}\frac{y}{x-\xi} \quad (5)$$

where

$$\gamma = \begin{cases}\gamma_a & 0 < \xi < 1\\ \gamma_j & 1 < \xi < \infty\\ 0 & -\infty < \xi < 0\end{cases}$$

The Fourier transform with its convolution theorem is used to obtain the interference potential φ_i, from Eqs. (1, 2, and 5). The solution yields the interference potential, hence, the up-

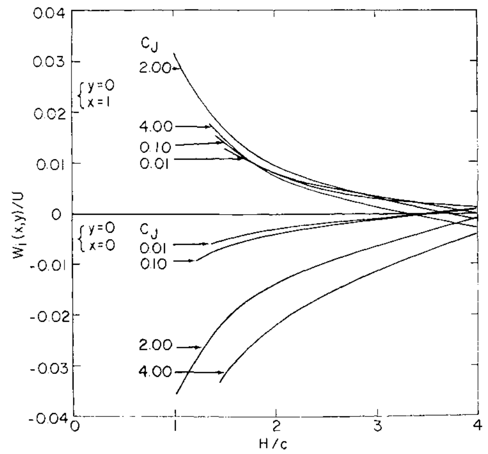

Fig. 4 Upwash interference as a function of H/c for a jet-flap wing with various values of C_J at $\alpha = \tau = 5.73°$ and $P = 0.6$.

wash velocity, expressed as

$$\frac{w_i}{U} = \frac{1}{U}\frac{\partial\varphi_i}{\partial y} = \frac{1}{2\pi}\int_0^\infty \gamma(\xi)d\xi \times \int_0^\infty \frac{1 - e^{-h_q} + kq\,e^{-h_q}}{\sinh qh + kq\cosh qh}\cosh qy \sin q(x-\xi)dq \quad (6)$$

where $h = H/c$ and $k = K/c$.

Results and Discussion

The upwash interference w_i may be written in the form $w_i/U = (\partial w_i/\partial\alpha)\alpha + (\partial w_i/\partial\tau)\tau$ where $\partial w_i/\partial\alpha$ and $\partial w_i/\partial\tau$ are functions of C_J, k, h, x, and y. The numerical results of $(\partial w_i/\partial\alpha)$ $(\partial w_i/\partial\tau)$ at the tunnel centerline are plotted in Fig. 2 for $C_J = 2.0$, $h = 1.5$ at various wall porosities. The smallest interference upwash is the closed tunnel case that is compatible with the two-dimensional lift interference on a small chord wing in a slotted tunnel as in Ref. 6.

The upwash distributions off the tunnel centerline are shown in Fig. 3 by $(\partial w_i/\partial\alpha)$ and $(\partial w_i/\partial\tau)$ terms. The results indicate that the upwash gradient in y direction is very small. The effect of the relative tunnel height H/c at various jet momentum coefficients C_J is shown in Fig. 4. It is as expected that the upwash interference will be decreased by decreasing C_J and/or increasing H/c.

The over-all corrections to the measured wing incidence, jet momentum coefficient, wing lift, and thrust coefficient may be calculated[7] by the utilization of the upwash interference. The present theory may be extended to seek an optimum slotted-wall configuration and the proper model location in the tunnel as well as the three-dimensional jet-flap wing case.

References

[1] Lo, C.-F. and Binion, T. W., "A V/STOL Wind Tunnel Wall Interference Study," *Journal of Aircraft*, Vol. 7, No. 1, Jan.-Feb. 1970, pp. 51–57.

[2] Lo, C.-F., "Wind Tunnel Boundary Interference on a V/STOL Model," to be published in *Journal of Aircraft*, Feb. 1971.

[3] Lo, C.-F., "Test Section for a V/STOL Wind Tunnel," *Journal of Aircraft*, Vol. 7, No. 4, July-August 1970, pp. 380–382.

[4] Williams, J. and Butler, S. F., "Experimental Methods for Testing High Lift BLC and Circulation Control Models," *Boundary Layer and Flow Control*, Vol. 1, edited by G. V. Lackmann, Pergamon Press, New York, 1961, p. 390.

[5] Spence, D. A., "The Lift Coefficient of a Thin Jet Flapped Wing," *Proceedings of the Royal Society (London)*, Series A, Vol. 238, No. 121, Dec./Jan. 1956–1957, pp. 46–68.

[6] Pindzola, M. and Lo, C. F., "Boundary Interference at Subsonic Speeds in Wind Tunnels with Ventilated Walls," AEDC-

TR-69-47, May 1969, Arnold Engineering Development Center, Tenn.

[7] Maskell, E. C., "The Interference on a Three-Dimensional Jet-Flap Wing in a Closed Wind Tunnel," R and M 3219, 1961, Aeronautical Research Council.

[8] Chen, C. F. and Mears, J. W., "Experimental and Theoretical Study of Mean Boundary Conditions at Perforated and Longitudinally Slotted Wind Tunnel Walls," AEDC-TR-57-20, Dec. 1957, Arnold Engineering Development Center, Tenn.

Spanwise Distribution of Induced Drag in Subsonic Flow by the Vortex Lattice Method

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Nomenclature

A = aspect ratio
C_{Di} = total induced drag coefficient of surface
C_{DiV} = total induced drag coefficient given by Vortex Lattice Method
C_{Diw} = total induced drag given by wake integral
C_L = total lift coefficient of surface
c = chord length of strip
$\bar{c}$ = average chord length of surface
c_{di} = induced drag coefficient of strip
c_l = lift coefficient of strip
D_d = normalwash factor at bound vortex
D_c = normalwash factor at control point
d = induced drag coefficient of box
e = semiwidth of strip
M = Mach number
p = pressure coefficient of box
S = reference area
s = semispan of surface
w_b = dimensionless normalwash at bound vortex
w_c = dimensionless normalwash at control point
x,y,z = Cartesian coordinates
α = angle of attack
α_i = induced incidence at bound vortex
η = dummy spanwise coordinate

Introduction

GARNER[1] has discussed induced drag and its spanwise distribution in incompressible flow. Following Multhopp,[2,3] we may write the induced drag coefficient as

$$C_{D_i} = \frac{1}{2S}\int_{-s}^{s} c_l c \alpha_i dy \qquad (1)$$

where the so-called induced incidence is

$$\alpha_i = \frac{1}{8\pi}\int_{-s}^{s}\frac{1}{y-\eta}\frac{d}{d\eta}(c_l c)d\eta \qquad (2)$$

Garner has shown that for sweptback wings, the quantity $c_l c \alpha_i$ bears no resemblance to the spanwise distribution of induced drag as suggested by Robinson and Laurmann[4] [Eq. (3.3,43)]. This Note reviews the calculation of the spanwise distribution of induced drag by the Vortex Lattice Method (VLM) and presents some comparisons with results of other methods.

The VLM in its modern form avoids the use of pressure loading functions and has been developed independently by several investigators including Rubbert[5] (who also considered the induced drag calculation), Dulmovits,[6] Hedman,[7] and Belotserkovskii.[8] In the application of this method, a division of the surface(s) into small trapezoidal elements (boxes) arranged in strips parallel to the freestream is made so that surface edges, fold lines, and hinge lines lie on box boundaries, as shown in Fig. 1. A horseshoe vortex is placed on each of the boxes such that the bound vortex of the horseshoe system coincides with the quarter-chord line of the box. The surface boundary condition is a prescribed normalwash (e.g., downwash) applied at the control point of each box. The control point is centered spanwise on the three-quarter chord line of the box. This choice of control point location has been shown by James[9] to be optimum for two-dimensional flow and results in a high degree of accuracy for three-dimensional flow. The influences of all the k vortices are summed for each control point j to obtain the total dimensionless normalwash w_c at the control point

$$w_{cj} = \sum_k D_{cjk} p_k \qquad (3)$$

where D_c is the three-quarter chord control point normalwash factor (given, e.g., by Hedman,[7] Appendix 3), and p is the pressure coefficient at the center of the bound vortex. Writing Eq. (3) for all control points j leads to a system of simultaneous equations whose solution for a prescribed distribution of normalwash leads to the desired distribution of pressure coefficients.

We now introduce the normalwash factor D_b that relates the normalwash w_b at the center of the bound vortex to the pressure coefficient of each horseshoe vortex. It is similar to D_c except that the normalwash induced by a bound vortex on itself is zero. Then, the induced drag coefficient d of box j is given by the Kutta-Joukowski Law to be

$$d_j = W_{bj} p_j \qquad (4)$$

where

$$W_{bj} = \sum_k D_{bjk} p_k \qquad (5)$$

The spanwise distribution of induced drag is found by summing the box drag coefficients along each strip, noting that the box drag coefficient is based on the local box area. The total induced drag coefficient is found by summing the drag over all strips on the surface and noting that the strip drag coefficient is based on the strip area.

Applications

The first application is to a rectangular wing with aspect ratio $A = 2.0$ in incompressible flow. A variety of aerodynamic idealizations of the planform into equal size rectangular boxes was investigated. Convergence of the total drag was obtained using 100 boxes from 5 chordwise and 20 spanwise divisions. However, the converged value of the induced drag was too low: $C_{D_i}/C_L^2 = 0.155375$, which is less than $1/\pi A = 0.159155$ by 2.43%.

The tendency of the VLM to predict low values of induced drag has already been observed by Rubbert.[5] This tendency remains to be explained, as do other aspects of the method,[9] but it suggests that a correcting scale factor be introduced into the spanwise distribution of induced drag that is based on the cross-flow energy in the wake (i.e., the total induced drag). The correcting scale factor is defined as the ratio $C_{D_{iw}}/C_{D_{iV}}$ where $C_{D_{iw}}$ is the wake integral given by Eq. (1) and $C_{D_{iV}}$ is the VLM result.

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